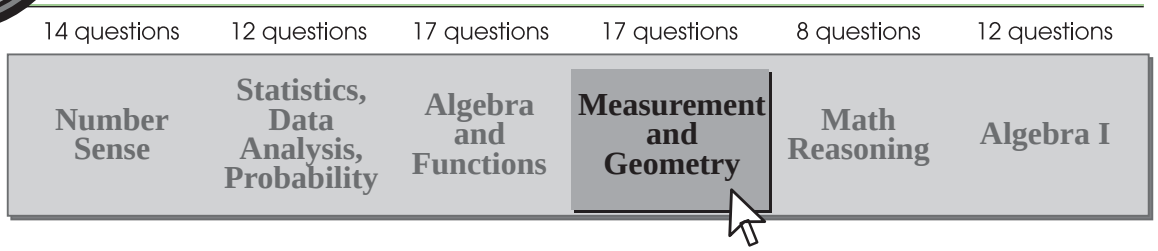
Measurement and Geometry



Seventeen of the 80 CAHSEE math questions are based on 10 selected standards from the grade 7 Measurement and Geometry strand.

WHAT DO THE MEASUREMENT AND GEOMETRY STANDARDS ASK ME TO DO?

The CAHSEE Measurement and Geometry questions will ask you to:

- convert measurements and rates from one measuring system to another
- use information from scale drawings
- know the effect of scaling on length, perimeter, area, and volume
- translate and reflect a shape drawn on a coordinate system
- know the Pythagorean theorem and its converse, and how and when to use each
- know that congruent objects have the same shape and size
- use lengths of an object to calculate the object's area, surface area, or volume

Specifically, you need to know how to calculate each of the following items:

- perimeter of a polygon (add up the lengths of the sides)
- circumference of a circle ($C = \pi d$ where d is the diameter)
- area of a parallelogram ($A = bh$ where b is base and h is height; the formula $A = bh$ also applies for finding the area of a rectangle because rectangles are just special kinds of parallelograms.)
- area of a triangle $\left(A = \frac{1}{2}bh\right)$
- volume of a rectangular solid ($V = lwh$ where l is length, w is width, and h is height)

Note: The formulas above are not provided on the exam, but all other formulas will be provided for you.

Vocabulary

The following words have appeared previously on the CAHSEE. If any of these words are unfamiliar to you, look them up in the CAHSEE Math Vocabulary list in the appendix at the back of this Study Guide, or check with your math teacher.

- area
- circle
- circumference
- congruent
- diameter
- hypotenuse
- parallel
- parallelogram
- perimeter
- radius
- surface area
- trapezoid
- volume

WHY ARE MEASUREMENT AND GEOMETRY IMPORTANT?

The mathematics from the Measurement and Geometry strand is used in architecture, landscaping, computer graphics, and the arts—and is also a foundation for calculus and other mathematics. The "anchor problem" for this strand, *Paving a Playground*, is from the building and construction trades and involves the use of many of the standards from this strand. But before we try *Paving a Playground*, let's first look at some sample CAHSEE questions, with answers, for this strand.

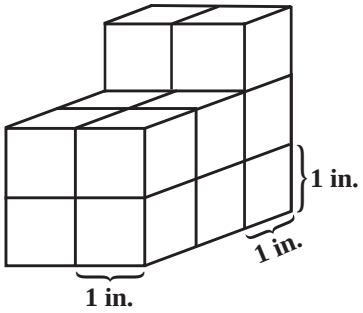
HOW WILL THE CAHSEE TEST MY KNOWLEDGE OF MEASUREMENT, USING MEASUREMENT, AND GEOMETRY?

The CAHSEE tests 10 of the 13 grade 7 standards from the Measurement and Geometry strand. Let's start by looking at 4 of these standards and the actual CAHSEE questions based on them. Each box that follows contains one of the standards, a sample question based on that standard, and a solution with explanation.

7MG2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects. [2 questions]

Sample CAHSEE Question

One-inch cubes are stacked as shown in the drawing below.



What is the <u>total</u> surface area?

A 19 in.^{2}

B 29 in.^{2}

C 32 in.^{2}

D 38 in.^{2}

M02812

Mathematical Solution

- Add all faces,
  $4+7+7+6+6+4+2+2=38$
- Therefore, the correct answer is **D**.

Descriptive Solution

Did you think the answer was 14? If so, you found the *volume* of this solid—it takes 14 cubes to build, so the volume is 14 cubic units. But this problem calls for surface area. What is surface area? If you put a solid object in water, the surface area of the object is the part that gets wet—the area of the outside surface. To find the total surface area of the solid above, you need to count up the number of square inches it takes to cover the outside, including the parts not visible in the picture. This object has several plane surfaces. Let's list the surfaces and the area of each: front, 4; right side, 7; left side (you don't see this one), 7; back, 6; bottom (you don't see this either), 6; top front, 4; top back, 2; and, finally, the front of the top two cubes, 2. Add these up and you get the total surface area: $4+7+7+6+6+4+2+2=38$ square inches. So the correct answer is **D**.

7MG2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and volume is multiplied by the cube of the scale factor. [1 question]

Sample CAHSEE Question

Bonni has two similar rectangular boxes. The dimensions of box 1 are twice those of box 2. How many times greater is the volume of box 1 than the volume of box 2?

A 3

B 6

C 8

D 9

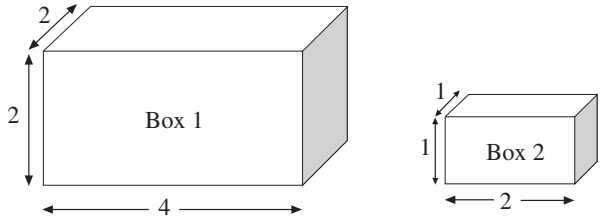
M21061

Mathematical Solution

- $V_{Box\ 1} = l \bullet w \bullet h$
- $V_{Box\ 2} = 2l \bullet 2w \bullet 2h$
  $= 2 \bullet 2 \bullet 2 \bullet l \bullet w \bullet h$
  $= 8 \bullet l \bullet w \bullet h$
- Therefore, the correct answer is **C**.

Descriptive Solution

To answer this question, picture two rectangular boxes, one with dimensions that are twice those of the other:



For this problem, imagine that box 2 has a length of 2, a width of 1, and a height of 1. The problem states that box 1 has dimensions twice those of box 2, so box 1 must have a length of 4, a width of 2, and height of 2.

The volume of each box can be found by multiplying its length by its width by its height $(V = lwh)$. Using this formula shows that the volume of box 1 is 16 $(V = 4 \bullet 2 \bullet 2)$ and the volume of box 2 is 2 $(V = 2 \bullet 1 \bullet 1)$. To determine how many times greater the volume of box 1 is, divide its volume by the volume of box 2, $\left(\frac{16}{2} = 8\right)$. Therefore, the correct answer is **C**.

Measurement Geometry

7MG3.2 Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections. [2 questions]

Sample CAHSEE Question

The points $(1, 1), (2, 3), (4, 3)$, and $(5, 1)$ are the vertices of a polygon. What type of polygon is formed by these points?

A Triangle

B Trapezoid

C Parallelogram

D Pentagon

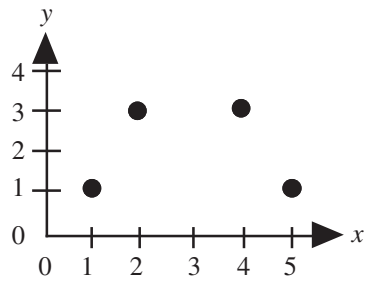
M02718

Mathematical Solution

- The correct answer is **B**. Please refer to the next column for a description of the solution.

Descriptive Solution

You'll want to plot these points on a grid to see what shape is formed. For each point, the first coordinate (x-coordinate) tells how far across to go, while the second coordinate (y-coordinate) tells how far up or down.

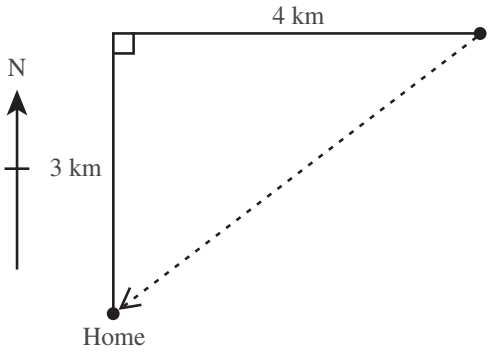


If you imagine these points connected in order with straight lines, you can see the correct answer must be **B**, trapezoid.

7MG3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement. [2 questions]

Sample CAHSEE Question

The club members hiked 3 kilometers north and 4 kilometers east, but then went directly home as shown by the dotted line. How far did they travel to get home?



A 4 km

B 5 km

C 6 km

D 7 km

M00120

Mathematical Solution

- Use the Pythagorean theorem, $a^2 + b^2 = c^2$.
- $3^2 + 4^2 = c^2$
- $9 + 16 = c^2$
- $25 = c^2$
- $\sqrt{25} = c$
- $5 = c$
- Therefore, the correct answer is **B**.

Descriptive Solution

The correct answer is **B**. Do you notice that the diagram shows a right triangle? The dashed line is the *hypotenuse*—the longest side. The other two sides which form the right angle, labeled 3 km and 4 km, are the *legs*. For all right triangles, the Pythagorean theorem says: *The sum of the squares of the legs equals the square of the hypotenuse*. In the figure above, the sum of the squares of the legs is $3^2 + 4^2 = 9 + 16 = 25$. Therefore, the hypotenuse is the square root of 25, which is 5.

USING MEASUREMENT AND GEOMETRY STANDARDS IN A REAL-LIFE SITUATION

To help you get the "big picture," following are seven Measurement and Geometry standards that are illustrated by an anchor problem called *Paving a Playground*; you might encounter problems such as this after high school. Even though the CAHSEE doesn't include problems with many calculations like this one, you might find it easier to remember one large problem (an "anchor problem"), in which many of the skills are combined, rather than trying to recall how to do each of the standards individually.

Try to do this problem before you look at its solution on the following pages.

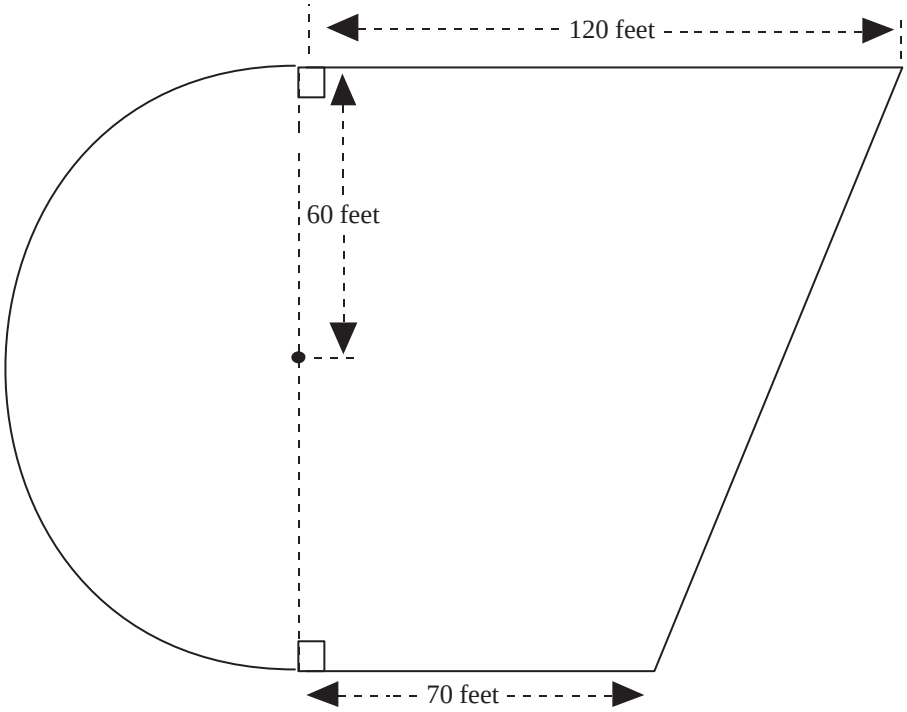
Paving a Playground

You work for a paving company and need to give a school a cost estimate for paving the playground and putting a concrete border around its perimeter. A scale drawing of the playground is shown below.

The cost (labor and materials) for the pavement is $54 per square yard.

The cost (labor and materials) for the concrete border is $18 per linear foot.

What's your estimate?

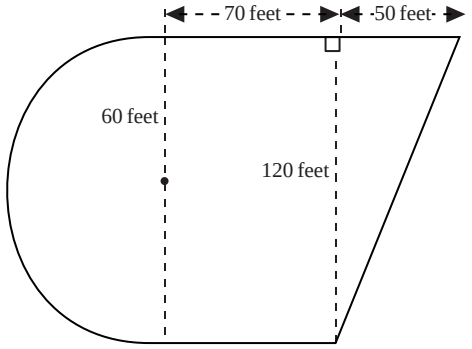


Paving a Playground Solution and Standards

7MG1.2 Construct and read drawings and models made to scale. [1 question]

To begin solving this problem, you'll first need to look at the diagram, read the lengths given, and make decisions about the missing lengths. Let's begin.

Do you see the semicircle, the rectangle, and the triangle? You can use what you know about these shapes plus the numbers given in the scale drawing to find the following lengths: the radius of the circle, and the length and width of the rectangle as shown:



Step 1: Determine the length of the playground's concrete border.

7MG3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement. [2 questions]

We can use the Pythagorean theorem to find the side of the triangle opposite the right angle (the hypotenuse). The Pythagorean theorem says that for a right triangle, the sum of the squares of the legs gives the square of the hypotenuse. In this figure, the legs are 50 and 120, so you would apply the theorem: $120^2 + 50^2 = 14{,}400 + 2{,}500 = 16{,}900$, which is the square of the hypotenuse. So the square root of 16,900 will be the length of the hypotenuse, 130 feet.

Next, you can find the length of the semicircular edge by using the formula for the circumference of a circle. A circle with a radius of 60 feet will have a circumference of $2\pi r$, where $\pi \approx 3.14$. $2\pi r = 2(3.14)60 = 376.8$ ft. But the playground's perimeter includes only half the circumference of the circle, which is 188.4 feet.

7MG2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms and cylinders. [3 questions]

Now you can add up the pieces to find the length of the playground's entire perimeter:

$$50 + 130 + 188.4 + 70 + 70 = 508.4 \text{ feet}$$

Step 2: Find the area of the playground by calculating the areas of the triangle, rectangle, and semi-circle.

Area of triangle is $\frac{1}{2}(50)(120) = 3{,}000$ square feet.

Area of rectangle is $(70)(120) = 8{,}400$ square feet.

Area of semicircle is $\frac{1}{2}\pi(60)^2 = 5{,}652$ square feet.

The sum of these three areas is the total area of the playground to be paved, 17,052 square feet.

Step 3: Figure out the cost of the pavement.
Let's go back to the original problem. What are you asked to find? You need to estimate the cost of paving the playground and its concrete border. Do you see that the cost of pavement and the concrete border are given as rates per unit? Pavement is $54 per square yard, and the border is $18 per linear foot.

Although the cost of pavement is given per square yard, we have calculated the area in square feet! We need to change the square feet into square yards. To do this you will need to use the fact that it takes 9 square feet to make 1 square yard. The area in square feet (17,052) divided by 9 will give the converted area: 1,895 square yards. Finally, you have to multiply the 1,895 square yards by the cost of $54 per square yard to get the final cost of the pavement: $102,330.

7MG1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer. [2 questions]

7MG1.1 Compare weights capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters). [2 questions]

Step 4: Figure out the cost of the border.
The only thing left to do is to find the cost of the border. You just need to multiply the perimeter, 508.4 feet, by $18 per linear foot.

$508.4(\$18) = \$9{,}151.$

7MG2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1\ ft^2] = [144\ in^2]$, 1 cubic inch is approximately 16.38 cubic centimeters. $[1\ in^3] = [16.38\ cm^3]$). [1 question]

Step 5: Determine the total cost estimate.
If you add the two money amounts together, $102,330 + $9,151, you will have a very good estimate for the work to be done by the paving company: $111,481 (nearest dollar).

Because this is an estimate, you may have rounded numbers off differently and found an estimate close to this. Did you get an estimate between $110,000 and $120,000?

7MG3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures. [1 question]

Paving a Playground—Again!
Suppose your company must pave another playground like this one. Could you use the same cost estimate? You could if the two playgrounds were congruent—if both had exactly the same shape and same size.

In order to solve this big problem, you used the math in 7 of the Geometry and Measurement standards. Now you are ready to answer the questions in the next section and then check your answers using the answer key provided in the appendix at the back of this Study Guide.

(Note: The CAHSEE questions used as examples throughout this Study Guide and in the following sample questions were used on prior CAHSEEs. These items will not be used in future CAHSEEs.)

ADDITIONAL MEASUREMENT AND GEOMETRY SAMPLE QUESTIONS

1. A boy is two meters tall. About how tall is the boy in feet (ft) and inches (in)? (1 meter ≈ 39 inches.)

A 5 ft 0 in

B 5 ft 6 in

C 6 ft 0 in

D 6 ft 6 in

M02044

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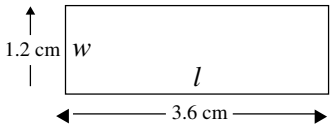
2. The actual width (w) of a rectangle is 18 centimeters (cm). Use the scale drawing of the rectangle to find the actual length (l).



A 6 cm

B 24 cm

C 36 cm

D 54 cm

M02087

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3. Beverly ran six miles at the speed of four miles per hour. How long did it take her to run that distance?

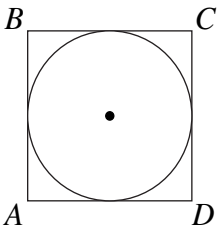
A $\frac{2}{3}$ hr

B $1\frac{1}{2}$ hrs

C 4 hrs

D 6 hrs

M02041



4. In the figure above, the radius of the inscribed circle is 6 inches (in). What is the perimeter of square *ABCD*?

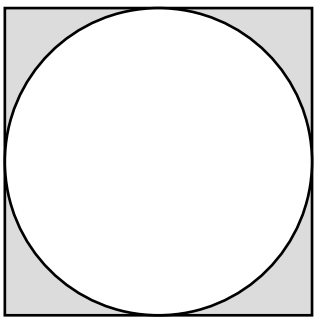
A 12π in

B 36π in

C 24 in

D 48 in

M02236



10 feet

5. The largest possible circle is to be cut from a 10-foot square board. What will be the approximate area, in square feet, of the remaining board (shaded region)? ($A = \pi r^2$ *and* $\pi \approx 3.14$)

A 20

B 30

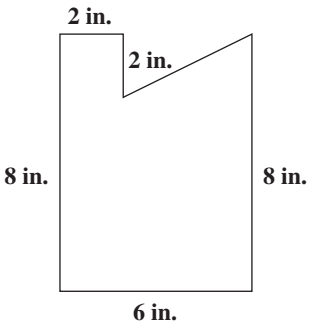
C 50

D 80

M00404

6. **A right triangle is removed from a rectangle as shown in the figure below. Find the area of the remaining part of the rectangle.**

$\left(\text{Area of a triangle} = \frac{1}{2}bh\right)$



A 40 in.^{2}

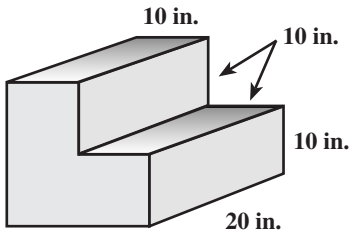
B 44 in.^{2}

C 48 in.^{2}

D 52 in.^{2}

M02093

7. **The short stairway shown below is made of solid concrete. The height and width of each step is 10 inches (in.). The length is 20 inches.**



What is the volume, in cubic inches, of the concrete used to create this stairway?

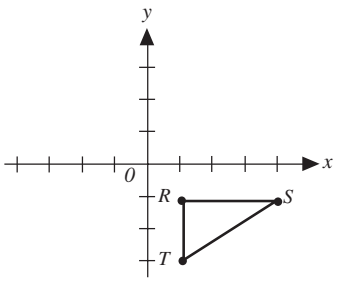
A 3000

B 4000

C 6000

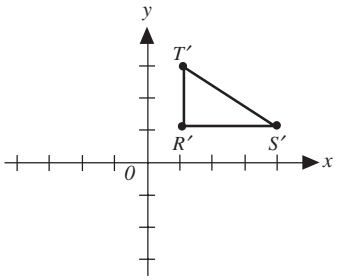
D 8000

M02990

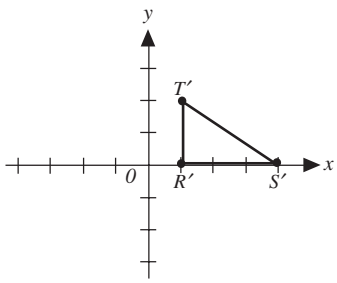


8. Which of the following triangles $R'S'T'$ is the image of triangle RST that results from reflecting triangle RST across the y-axis?

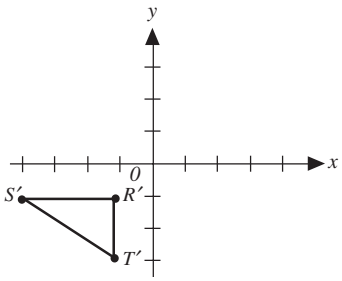
A



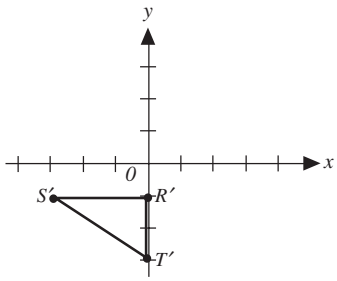
C



B



D



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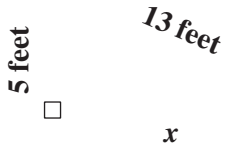
9. What is the value of x in the right triangle shown below?



A 8 feet

B 12 feet

C 18 feet

D 23 feet

M03181